ECE 720, Design Project 1, Fall 2005
Eigenvector Assignment for a Linearized Aircraft Model

Due Wednesday, October 19, 7:20 p.m.

A. Introduction

This project is to be an individual effort for each student. All work must be your own. Any questions about the project should be addressed to Dr. Beale.

A linearized model of the lateral dynamics of an aircraft is described by a continuous-time linear state-space model with 2 inputs, 2 outputs, and 4 states. The model is given by[1]

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad\qquad y(t) = Cx(t) + Du(t) \tag{1}$$

$$A = \begin{bmatrix} -10 & 0 & -10 & 0 \\ 0 & -0.7 & 9 & 0 \\ 0 & -1 & -0.7 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 20 & 2.8 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.15 \end{bmatrix} \tag{3}$$

where the state vector $x \in \Re^4$ is defined as $x_1 = p$ (roll rate, rad/sec), $x_2 = r$ (yaw rate, rad/sec), $x_3 = \beta$ (sideslip angle, rad), and $x_4 = \phi$ (roll angle, rad). The control vector $u \in \Re^2$ is defined as $u_1 = \delta_a$ (aileron angle, rad) and $u_2 = \delta_r$ (rudder angle, rad).

The closed-loop system will be achieved through constant full state feedback with the control law

$$u(t) = -K_c x(t) \tag{4}$$

The control gain matrix K_c will be chosen to place the closed-loop eigenvalues and eigenvectors. The closed-loop eigenvalues will be fixed throughout the project; different closed-loop eigenvectors will be used. The specified locations for the closed-loop eigenvalues are

$$\lambda_{closed-loop} = [-1.5, \ -3, \ -7, \ -9] \tag{5}$$

[1] W.L. Brogan, *Modern Control Theory*, Third Edition, Prentice Hall, Englewood Cliffs, NJ, 1991, page 390.

B. *Tasks to be Performed*

1) Using the MATLAB *place* function, compute the gain matrix $K_{c-place}$ that places the closed-loop eigenvalues at the required locations. Verify that the eigenvalues are placed correctly, determine the closed-loop eigenvectors, and investigate the orthogonality of the eigenvectors. With the specified initial conditions for the state vector, simulate the closed-loop system to determine the responses of the state variables. Determine the closed-loop transmission zeros for the system, $K_{c-place}\left(sI - A_{CL}\right)^{-1}B$, where $A_{CL} = A - BK_{c-place}$. The MATLAB function *tzero* can be used for this.

2) Using the output results in part 1 as a standard, compute at least two additional gain matrices using the eigenvector assignment method discussed in class[2]. One of these gain matrices should be chosen to give “very bad” performance. The other gain matrix should give comparable or better performance than that obtained with the *place* function. You may choose your own method of evaluating the quality of the performance, as long as it is reasonable. For each of your gain matrices, determine the closed-loop transmission zeros, investigate the orthogonality of the eigenvectors, and simulate the closed-loop system with the specified initial conditions.

3) Write a report that compares your various designs with that obtained from the MATLAB *place* function. Discuss the performance obtained from the simulation results in terms of the orthogonality of eigenvectors, the locations of the closed-loop transmission zeros, and the norms of the gain matrices. Discuss how you chose the eigenvector placements, and justify your selection of the performance evaluation criterion. Include plots to illustrate the various responses achieved using the different gain matrices.

[2] *Multivariable Control, Placement of Eigenvectors*, G.O. Beale, pdf file located on my Robust and Multivariable Control examples web page at http://ece.gmu.edu/~gbeale/ece_720/examples_720.html.