ECE 720 Design Project 2, Fall 2005 Robust Stability and Performance for a Ship Model

Due Wednesday, December 14, 7:20 p.m.

A. Introduction

The open-loop transfer function for a Mariner-class cargo ship under nominal conditions is given by

$$G_{p0}(s) = \frac{\psi(s)}{\delta_r(s)} = \frac{K_0\,(s - z_0)}{s\,(s - p_1)\,(s - p_2)\,(s - p_3)} = \frac{-3.7424 \cdot 10^{-3}\,(s + 5.3879 \cdot 10^{-2})}{s\,(s + 8.4688 \cdot 10^{-3})\,(s + 1.2870 \cdot 10^{-1})\,(s + 0.3333)} \tag{1}$$

where the output signal $\psi(t)$ is the heading (yaw) angle of the ship, and the input signal $\delta_r(t)$ is the commanded rudder angle. The pole p_3 at $s = -0.3333$ models the actuator that converts the commanded rudder angle into the actual rudder angle. The gain K_0, the zero at z_0, and the two poles at p_1, p_2 model the ship's dynamics from actual rudder angle to yaw rate. The pole at $s = 0$ represents the integration from yaw rate to yaw angle. The model in (1) will be considered as the nominal model for this project.

Two possible perturbations to the nominal ship model are

$$G_{p1}(s) = \frac{-1.4970 \cdot 10^{-2}\,(s + 2.6940 \cdot 10^{-2})}{s\,(s + 1.6938 \cdot 10^{-2})\,(s + 1.2870 \cdot 10^{-1})\,(s + 0.3333)} \tag{2}$$

$$G_{p2}(s) = \frac{-9.3560 \cdot 10^{-4}\,(s + 1.0776 \cdot 10^{-1})}{s\,(s + 4.2344 \cdot 10^{-3})\,(s + 1.2870 \cdot 10^{-1})\,(s + 0.3333)} \tag{3}$$

Perturbations to the nominal model will be represented in the form of multiplicative uncertainty, that is,

$$G_p(s) = G_{p0}(s)\,[1 + w_I(s)\Delta_I(s)]\ , \qquad |\Delta_I\,(j\omega)| \le 1\ , \forall\omega \tag{4}$$

where $G_p(s)$ represents the entire set of system models that must be stabilized. The perturbed transfer functions given in (2) and (3) must fall within the family of models defined by $G_p(s)$ in (4). Thus, $w_I(s)$ must be such that $G_{p0}(s)$, $G_{p1}(s)$, and $G_{p2}(s)$ are all elements of the family for which robust stability and robust performance are achieved.

A phase lead compensator that stabilizes each of the three system models is given by

$$G_c(s) = \frac{-0.43619\,(s + 0.02)}{(s + 5.5299 \cdot 10^{-2})} \tag{5}$$

This compensator will be used for part of your analysis in the project. The compensator in (5) is equivalent to a Proportional + Derivative (PD) controller of the form

$$G_c(s) = -\left(K_p + \frac{K_d \cdot s}{\tau s + 1}\right) = -\left(0.15776 + \frac{5.035 \cdot s}{18.083 \cdot s + 1}\right) \tag{6}$$

B. *Tasks to be Performed*

1. Using the models in (2) and (3) as perturbations to the nominal model in (1), develop a stable transfer function $w_I(s)$ that overbounds the maximum relative perturbation

$$l_I\left(j\omega\right) = \max_{G_{pi}} \left| \frac{G_{pi}(j\omega) - G_{p0}(j\omega)}{G_{p0}(j\omega)} \right| \tag{7}$$

   at all frequencies. Relative magnitude plots for the two perturbed transfer functions relative to G_{p0} are shown in Fig. 1. These two plots must be overbounded by $\left|w_I\left(j\omega\right)\right|, \forall\omega$. You want the overbounding to be as tight as possible without having a transfer function of too high an order. A second-order or third-order transfer function for $w_I(s)$ should be sufficient.

   Using the nominal plant model (1), the compensator in (5), and your weighting function $w_I(s)$, determine if robust stability is achieved under the assumption that $\Delta_I(s)$ can be any stable function that satisfies the magnitude bound in (4). If robust stability is not achieved, include in your report how the plant models given in (2) and (3) can be stabilized by $G_c(s)$ but some plant models described in (4) are not stabilized by that compensator.

2. The specifications on the sensitivity function magnitude are:
   a. The bandwidth defined in terms of $\left|S\left(j\omega\right)\right|$ should be in the range $0.02 \le \omega_B \le 0.1$ rad/sec;
   b. The sensitivity magnitude should satisfy $\left|S\left(0\right)\right| \le 1 \cdot 10^{-5}$
   c. The sensitivity magnitude should satisfy $\max_{\omega}\left|S\left(j\omega\right)\right| = \left\|S\left(s\right)\right\|_{\infty} \le 1.5$.

   Develop a stable transfer function $w_P(s)$ such that $1/\left|w_P\left(j\omega\right)\right|$ satisfies the above three requirements, allowing $1/\left|w_P\left(j\omega\right)\right|$ to be used as an upper bound for $\left|S\left(j\omega\right)\right|$. Determine whether or not any of the sensitivity functions obtained from $L_0 = G_c G_{p0}$, $L_1 = G_c G_{p1}$, or $L_2 = G_c G_{p2}$ satisfies this upper bound. If robust stability has been achieved, determine if robust performance is also achieved.

3. Determine the generalized plant model P that corresponds to the mixed S/T optimization problem for this system.

4. Using Linear Quadratic Gaussian with Loop Transfer Recovery (LQG/LTR) or H_∞ methodologies, design a compensator that provides robust stability and robust performance for this system. If you use H_∞ and do not have access to the Robust Controls Toolbox, you may send the MATLAB code to me. I will execute the code and return the results to you.

5. Prepare a typed report that presents your analysis of the robust stability and robust performance of this system with the compensator given in (5). Include the same analysis for the compensator that you design. If robust stability or robust performance are not achieved with your design, discuss possible reasons for this. Include plots to illustrate the various aspects of your analysis, such as Bode plots and polar plots to indicate nominal stability, nominal performance, robust stability, and robust performance, and step response plots to indicate the connections between the time and frequency domains.

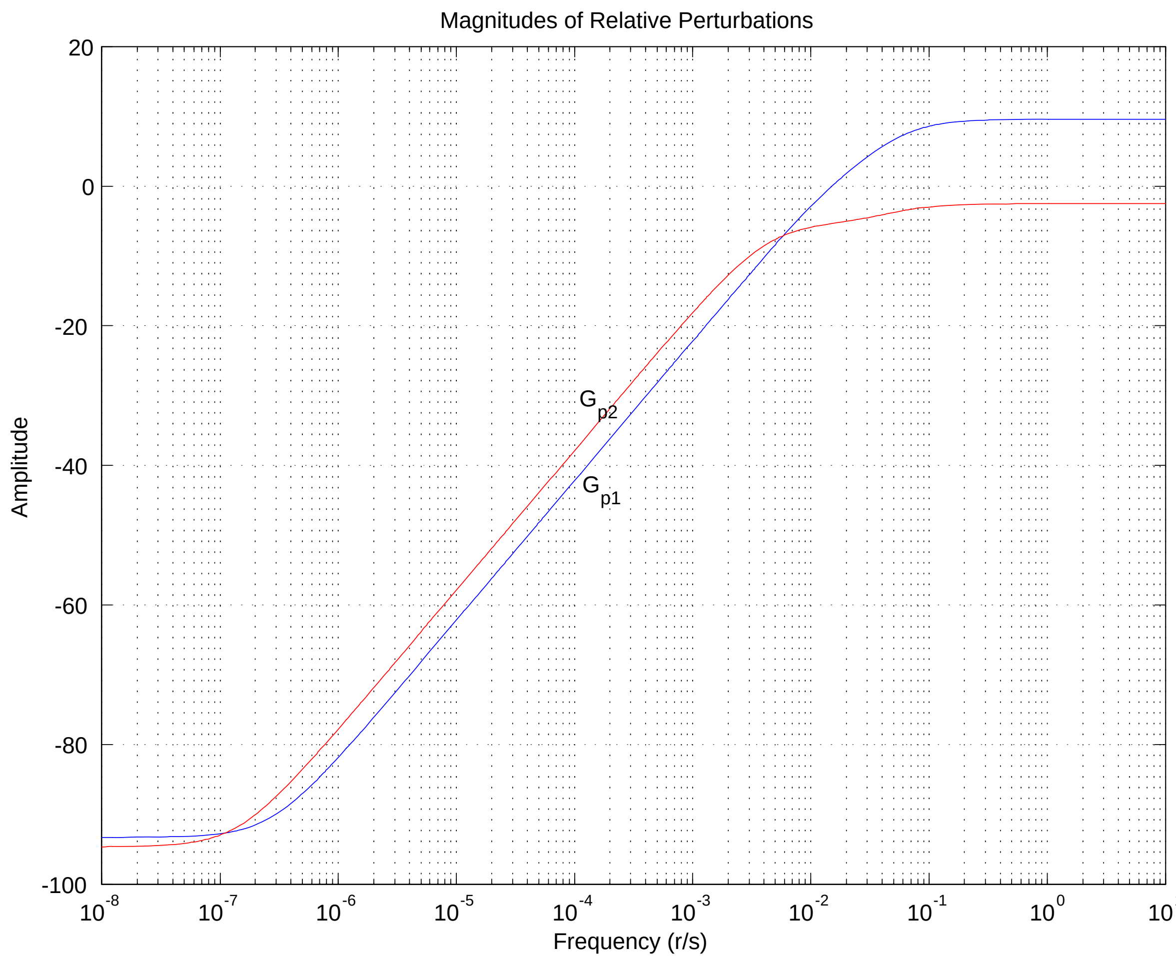

Fig. 1. Bode magnitudes of the perturbed system models relative to $G_{p0}(s)$.