ECE 720 Project #3, Fall 2005
Paper Review

Due Wednesday, December 14, 7:20 p.m.

Reference:

- K. Hayakawa, K. Matsumoto, M. Yamashita, Y. Suzuki, K. Fujimori, and H. Kimura, *Robust H^∞ Output Feedback Control of Decoupled Automobile Active Suspension Systems*, IEEE Transactions on Automatic Control, Vol. 44, No. 2, February 1999, pp. 392–396.

This project is to be an individual effort for each student. All work must be your own. Any questions about the project should be addressed to Dr. Beale.

1. The referenced paper deals with the design of a robust control system for an active suspension system of an automobile. The design method that is used is H_∞ with frequency-selective weighting functions to provide the proper robustness. Robust stability against unstructured uncertanities and good disturbance rejection are the goals.
2. The referenced paper is to be read for educational purposes and analyzed relative to its importance in the field of control and the topics covered in this course on robust control theory. The primary focus for your review of the paper should be Section IV, Design of Control System. Your review should include:
   a. how this paper fits into the material covered in class;
   b. the accuracy of the expressions in equations (9), (13)–(16);
   c. your estimate of the transfer function for the weighting function shown in Fig. 3 and the transfer function for the robustness function shown in Fig. 4 (dashed lines). The same robustness function appears in each of the three subplots of Fig. 4.
   d. Your opinion on the results presented in Section V of the paper in terms of the designed active suspension relative to conventional passive suspension.
3. A typed report is due on the day and time specified above. The report should provide a clear description of your review of the paper, focusing on the topics in item 2.

matrices solving the problem have been derived (Theorem 4.1). The problem of partial model matching with simultaneous stabilization for SISO systems has been studied separately (Section V-A), where it is shown that all controllable poles of the closed-loop system are affected by the degrees of freedom of the controller matrices. For this reason, only sufficient conditions have been established. Additionally, for SISO systems, the necessary and sufficient conditions for partial model matching with simultaneous regulation of the free response of the closed-loop system have been established (Section V-B). For MIMO systems the problem of partial model matching, via regular static measurement output feedback, has been solved in Section VI. Finally, the special case of partial model matching, called here the partial zeroing of the input–output map, has been studied in Section VII, wherein the necessary and sufficient conditions for the solution of the problem with simultaneous stabilizability have been established.

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Robust H^∞-Output Feedback Control of Decoupled Automobile Active Suspension Systems

Kisaburo Hayakawa, Kenichiro Matsumoto, Masashi Yamashita, Yoshiaki Suzuki, Kazuo Fujimori, and Hidenori Kimura

***Abstract*— H^∞-output feedback control is applied to the control of automobile active suspensions based on a dynamic model of the full vehicle. The output feedback control is desirable from the viewpoint of implementation in the sense that it reduces the number of measurements drastically compared with state feedback used currently in suspension control. In this paper, the authors show that a linearized model can be block-decoupled by a similarity transformation, which reduces the controller complexity significantly. The uncertainties of the vehicle model are properly taken into account in the derivation of the H^∞ controller. The controller is actually implemented in a commercial car, and the performance is evaluated both by simulations and experiments. The performance obtained proves to be quite satisfactory.**

***Index Terms*—Active suspension, automobile, decoupling, H^∞ control, output feedback.**

I. INTRODUCTION

The active control of automobile suspension is used to improve the ride comfort without sacrificing the body leveling and drivability. To accomplish these control objectives, which severely conflict with each other, the design of an active suspension controller must be based on the full understanding of the dynamic behaviors of the vehicle which are represented as complicated interactions among various modes corresponding to dynamics of the body and the wheels. Unfortunately, however, the present controllers of active suspensions which are commercially used are designed based on intuitive reasoning and heuristic tuning without taking into account the complicated interaction among the various modes of the body–wheels behaviors [1], [2]. The performances obtained by these conventional-type controllers are not regarded as satisfactory as expected. So far, there have appeared some papers on the application of linear-quadratic (LQ) and/or linear-quadratic-Gaussian (LQG) theory to active suspensions of a quarter-car model [3]–[5] and a half-car model [6], [7]. Moreover, Moran and Nagai [8] applied H^∞-control theory to active suspensions of a half-car model and they examined the robustness properties of the H^∞ controller and compared with those of a LQG controller about the variations in tire stiffness and car body mass. They showed that the H^∞ controller was more robust than the LQG one. But, only a few papers have dealt with the multivariable control of active suspensions based on a full dynamic model of the vehicle [9]–[12]. Perhaps, the paper by Yamashita *et al.* [13], [14] was the first contribution to the multivariable control of active suspensions which includes experimental evaluation of the performances through the actual implementation of the controller. They used the H^∞-

Manuscript received October 28, 1997. Recommended by Associate Editor, J. Sun.

K. Hayakawa is with Toyota Central Research and Development Laboratories, Inc., System Control Lab., Nagakute-cho, Aichi-gun, Aichi 480-11, Japan (e-mail: khaya@fldlab.tytlabs.co.jp).

K. Matsumoto is with Komatsu Ltd., Ageno, Hirakata 573, Japan.

M. Yamashita, Y. Suzuki, and K. Fujimori are with the Toyota Motor Corporation, Toyota, Aichi 471, Japan.

H. Kimura is with the Department of Mathematical Engineering and Information Physics Faculty of Engineering, University of Tokyo, Hongo, Bunkyou-ku, Tokyo 113, Japan.

Publisher Item Identifier S 0018-9286(99)00577-2.

control method and showed that the active suspension was capable of accomplishing the improvements of ride comfort and drivability [14] simultaneously, as well as obtaining sufficient robustness against various parameter variations and uncertainties. A weakness of their results was that they used the full state feedback which causes some difficulties in implementation because some of the states are not easy to measure.

In this paper, we propose a design method of active suspension control which is conceptually simpler and easier to implement than the one in [14]. It is based on the observation that the linear dynamical model of the full vehicle is intrinsically decoupled into two parts. More specifically, an appropriate selection of the state vector enables us to represent the model of the full vehicle in such a way that the heave and the pitch modes and the roll and the warp modes are decoupled completely. In other words, the transfer function matrix representing the heave and the pitch modes are decoupled from that representing the roll and the warp modes. We shall prove it based on the dynamical model of the full vehicle with 7 degrees of freedom which is derived based on the Kane's dynamics [15]. This result gives us a sound verification of the method of decoupling proposed by Malek and Hedrick [9].

Based on this observation, we implement robust H^∞ controllers to the block-decoupled systems. The main advantage of using the H^∞-control method is the enhancement of robustness when unmodeled dynamics, nonlinear behavior, and parameter variations exist in the actual system, as well as the possibility of frequency domain-shaping taking the human body sensitivity characteristic against vibration into account. The block decoupling reduces the controller complexity significantly and makes the output feedback effective. The simulations, as well as the experimental results, demonstrate the superiority of our method.

II. Modeling and Decoupling

In order to derive the model of an active suspension vehicle in a systematic way, we use the Kane's dynamics [15]. Consequently, the full vehicle model with 7 degrees of freedom including active suspensions is derived as the following state-space equations [16], [17]:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w \\ y &= C_2 x \end{aligned} \tag{1}$$

where the state x, the disturbance w, the input u, the controlled value z and the output y are expressed as

$$\begin{aligned} x &= [v_z \quad \theta \quad \varphi \quad z_{st1\sim 4} \quad \dot{z}_{wl\sim 4} \quad z_{rw1\sim 3} \quad q_{1\sim 4} \quad v_{1\sim 4}]^T \\ w &= [\dot{z}_{r1\sim 4} \quad z_{r1\sim 4} \quad a_x \quad a_y]^T \\ u &= [i_1 \quad i_2 \quad i_3 \quad i_4]^T \\ z &= [\dot{v_z} \quad \dot{\theta} \quad \dot{\varphi} \quad z_{sth} \quad z_{str} \quad z_{stp} \quad z_{stw}]^T \\ y &= [v_z \quad \theta \quad \varphi \quad z_{st1} \quad z_{st2} \quad z_{st3} \quad z_{st4}]^T \end{aligned} \tag{2}$$

where

- a_x body longitudinal force;
- a_y body lateral force;
- i_i current for the ith servo valve;
- θ body roll angular velocity;
- φ body pitch angular velocity;
- q_i flow rate of oil through the ith servo valve;
- v_i volume of oil through the ith servo valve;
- v_z body vertical velocity;
- z_{ri} road displacement at the ith tire;
- z_{rwi} ith tire deflection;
- z_{sti} ith suspension deflection;
- z_{sti} ith suspension deflection;
- z_{wi} ith wheel vertical displacement.

The latter portions of vector z and y are related as

$$\begin{bmatrix} z_{sth} \\ z_{str} \\ \cdots \\ z_{stp} \\ z_{stw} \end{bmatrix} = \hat{M} \begin{bmatrix} z_{st1} \\ z_{st2} \\ \cdots \\ z_{st3} \\ z_{st4} \end{bmatrix}, \; \hat{M} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}. \tag{3}$$

We call the matrix the $\hat{M}$ *mode transformation matrix,* which is used for the decoupling between the mode pairs in the next section. The subscripts h, r, p, and w denote *heave, roll, pitch*, and *warp modes*, respectively.

III. Decoupling

Applying the following similarity transformations:

$$\begin{aligned} \hat{x} &= \begin{bmatrix} 1 & & & & & 0 \\ & 1 & & & & \\ & & 1 & & & \\ & & & \hat{M} & & \\ & 0 & & & \ddots & \\ & & & & & \hat{M} \end{bmatrix} x, \\ \hat{w} &= \begin{bmatrix} 1 & & & 0 \\ & \hat{M} & & \\ & & 1 & \\ 0 & & & \hat{M} \end{bmatrix} w, \qquad \hat{u} = \hat{M} u \end{aligned} \tag{4}$$

to the system (1) where $\hat{M}$ is the mode transformation matrix given in (3), and rearranging x, w, u, z, and y, we can show the transformed system has the realization of the form

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}}_1 \\ \cdots \\ \dot{\hat{x}}_2 \end{bmatrix} &= \begin{bmatrix} A_1^D & 0 \\ \cdots & \cdots \\ 0 & A_2^D \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \cdots \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} B_{11}^D & 0 \\ \cdots & \cdots \\ 0 & B_{12}^D \end{bmatrix} \begin{bmatrix} \hat{w}_1 \\ \cdots \\ \hat{w}_2 \end{bmatrix} \\ &\quad + \begin{bmatrix} B_{21}^D & 0 \\ \cdots & \cdots \\ 0 & B_{22}^D \end{bmatrix} \begin{bmatrix} \hat{u}_1 \\ \cdots \\ \hat{u}_2 \end{bmatrix} \\ \begin{bmatrix} z_1 \\ \cdots \\ z_2 \end{bmatrix} &= \begin{bmatrix} C_{11}^D & 0 \\ \cdots & \cdots \\ 0 & C_{12}^D \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \cdots \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} D_{111}^D & 0 \\ \cdots & \cdots \\ 0 & D_{112}^D \end{bmatrix} \begin{bmatrix} \hat{w}_1 \\ \cdots \\ \hat{w}_2 \end{bmatrix} \\ \begin{bmatrix} y_1 \\ \cdots \\ y_2 \end{bmatrix} &= \begin{bmatrix} C_{21}^D & 0 \\ \cdots & \cdots \\ 0 & C_{22}^D \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \cdots \\ \hat{x}_2 \end{bmatrix} \end{aligned} \tag{5}$$

as shown in (6), at the bottom of the next page. The representation (5) implies that the transfer function matrix from $[\hat{u}_1 \;\; \hat{u}_2]$ to $[z_1 \;\; z_2]$ is of block diagonal form, as well as that of $[\hat{u}_1 \;\; \hat{u}_2]$ to the $[y_1 \;\; y_2]$. In other words, the input–output relation between $\hat{u}_1$ and z_1 which we call the *(heave, pitch)* system is decoupled from the input–output relation between $\hat{u}_2$ and z_2 which we call the *(roll, warp)* system.

IV. Design of Control System

A. Specification of Control System

We consider the following control objectives to design the control system of the automobile active suspension system:

1) reduction of the sensitivity from the disturbance w to the controlled value z of system (1) to improve both ride comfort and the body leveling;
2) the robustness of the stability and control performance of closed-loop system against the perturbation due to the model uncertainty.

For the sake of simplicity, the control design only for the (heave, pitch) system is explained because the same control design is used for the (roll, warp) system.

The (heave, pitch) system is expressed as follows:

$$\begin{aligned}\dot{\hat{x}}_1 &= A_1^D \hat{x}_1 + B_{11}^D \hat{w}_1 + B_{21}^D \hat{u}_1\\ z_1 &= C_1^D \hat{x}_1 + D_{111}^D \hat{w}_1\\ y_1 &= C_{21}^D \hat{x}_1.\end{aligned} \tag{7}$$

The controller is given by

$$\hat{u}_1 = K_1 y_1.$$

The transfer matrix for the (heave, pitch) system is defined as

$$\begin{bmatrix} z_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} G_{11}^1 & G_{12}^1 \\ G_{21}^1 & G_{22}^1 \end{bmatrix} \begin{bmatrix} \hat{w}_1 \\ \hat{u}_1 \end{bmatrix}. \tag{8}$$

For the control objective 1), the transfer matrix from the disturbance w_1 to the controlled value z_1 of the (heave, pitch) system is expressed as

$$z_1 = \Phi_1 \hat{w}_1, \qquad \Phi_1 = G_{11}^1 + G_{12}^1 K_1 (I - G_{22}^1 K_1)^{-1} G_{21}^1 \tag{9}$$

and the following cost function in terms of H$^\infty$ norm for control performance is considered:

$$\|W_1 \Phi_1\|_\infty < \gamma_1, \qquad \exists \gamma_1 > 0 \tag{10}$$

where W_1 is the weighting function for the (heave, pitch) system which is chosen as the cost criterion of control performance.

For the control objective 2), it is assumed that the uncertainty Δ of the model exists in the input channel as shown in Fig. 1. Then, from the *small gain theorem*, a sufficient condition for robust stability is derived as follows.

For the matrix R_1

$$R_1 = \mathrm{diag}(r^1), \qquad r^1 = [r_1^1 \quad r_2^1] \tag{11}$$

which satisfies the inequality

$$|\Delta_i^1(j\omega)| < |r_i^1(j\omega)| \qquad (i = 1, 2) \tag{12}$$

for the important region ω which corresponds to the bound with respect to the control performance. The system of (8) is robustly stable by means of K_1 if

$$\|R_1 K_1 (I - G_{22}^1 K_1)^{-1} G_{22}^1\|_\infty < 1. \tag{13}$$

Consequently, the control objectives 1) and 2) are satisfied by the following cost function:

$$\left\| \begin{matrix} \overline{W}_1 H_{z_1 \hat{w}_1}^1 & \overline{W} H_{z_1 p_1}^1 \\ H_{q_1 \hat{w}_1}^1 & H_{q_1 p_1}^1 \end{matrix} \right\|_\infty < 1 \tag{14}$$

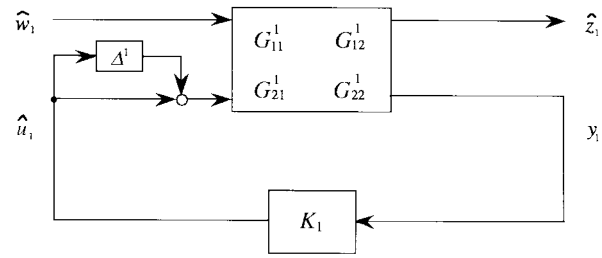

Fig. 1. Block diagram of the (heave, pitch) system and the uncertainty.

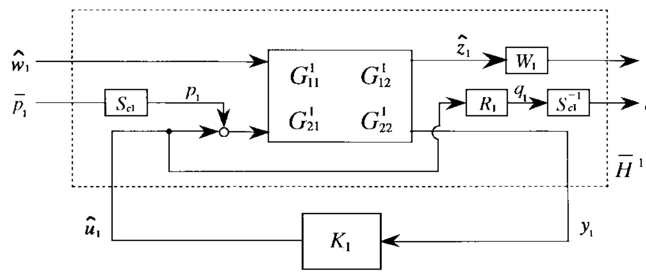

Fig. 2. Block diagram of the augmented plant of the (heave, pitch) system.

where $\overline{W}_1 = W_1/\gamma_1$ and the matrix H^1 denotes the transfer matrix which is expressed by the following equality:

$$\begin{bmatrix} z_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} H_{z_1 \hat{w}_1}^1 & H_{z_1 p_1}^1 \\ H_{q_1 \hat{w}_1}^1 & H_{q_1 p_1}^1 \end{bmatrix} \begin{bmatrix} \hat{w}_1 \\ p_1 \end{bmatrix} \tag{15}$$

where

$$\begin{aligned} H_{z_1 \hat{w}_1}^1 &= G_{11}^1 + G_{12}^1 K_1 (I - G_{22}^1 K_1)^{-1} G_{21}^1 = \Phi_1 \\ H_{z_1 p_1}^1 &= G_{12}^1 + G_{12}^1 K_1 (I - G_{22}^1 K_1)^{-1} G_{22}^1 \\ H_{q_1 \hat{w}_1}^1 &= R_1 K_1 (I - G_{22}^1 K_1)^{-1} G_{21}^1 \\ H_{q_1 p_1}^1 &= R_1 K_1 (I - G_{22}^1 K_1)^{-1} G_{22}^1 \end{aligned}$$

and the subscript p_1 and q_1 of the transfer matrix H^1 show the external input with respect to perturbation and the output which denotes the quantity of uncertainty, respectively, as shown in Fig. 2 that is the generalized plant taking (11) and (12) into account in Fig. 4. It is clear that (10) and (13) hold if (14) is satisfied and robust performance is guaranteed by bounding the norm of the nondiagonal matrix in (14).

A controller K_1 which satisfies (14) is apt to be conservative because the magnitude of control input $\hat{u}_1$ is severely bounded due to the requirement of reducing the H$^\infty$ norm of the transfer matrix $H_{q_1 \hat{w}_1}$.

$$\begin{aligned}
\hat{x}_1 &= [v_z \quad \varphi \quad z_{sth} \quad z_{stp} \quad \dot{z}_{wh} \quad \dot{z}_{wp} \quad z_{rwh} \quad z_{rwp} \quad q_h \quad q_p \quad v_h \quad v_p]^T \\
\hat{x}_2 &= [\theta \quad z_{str} \quad z_{stw} \quad \dot{z}_{wr} \quad \dot{z}_{ww} \quad z_{rwr} \quad q_r \quad q_w \quad v_r \quad v_w]^T \\
\hat{w}_1 &= [\dot{z}_{rh} \quad \dot{z}_{rp} \quad z_{rh} \quad z_{rp} \quad a_x]^T \\
\hat{w}_2 &= [\dot{z}_{rr} \quad \dot{z}_{rw} \quad z_{rr} \quad z_{rw} \quad a_y]^T \\
\hat{u}_1 &= [i_h \quad i_p]^T, \\
\hat{u}_2 &= [i_r \quad i_w]^T \\
z_1 &= [\dot{v}_z \quad \dot{\varphi} \quad z_{sth} \quad {}_{stp}]^T \\
z_2 &= [\dot{\theta} \quad z_{str} \quad z_{stw}]^T \\
y_1 &= [v_z \quad \varphi \quad z_{sth} \quad z_{stp}]^T \\
y_2 &= [\theta \quad z_{stp} \quad z_{stw}]^T
\end{aligned} \tag{6}$$

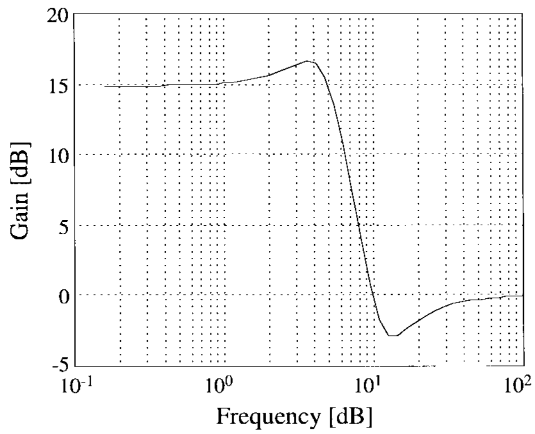

Fig. 3. Weighting function for the body accelerations.

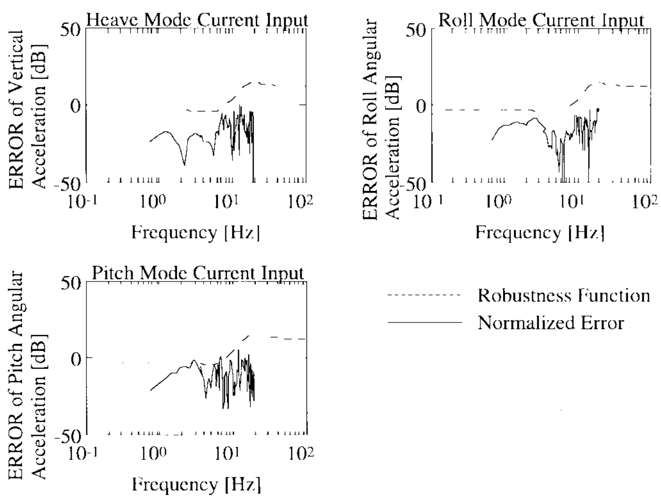

Fig. 4. Normalized errors of the body accelerations between the experimental vehicle and the model and their robustness functions.

Therefore, we consider the scaling of the input p_1 and the output q_1 as shown in Fig. 2 and to reduce the conservatism in the same line as μ-synthesis [18]. The following norm condition is considered instead of (14):

$$\left\| \begin{matrix} \overline{W}_1 H^1_{z_1 \hat{w}_1} & \overline{W}_1 H^1_{z_1 p_1} S_{c1} \\ S^{-1}_{c1} H^1_{q_1 \hat{w}_1} & S^{-1}_{c1} H^1_{q_1 p_1} S_{c1} \end{matrix} \right\|_\infty < 1. \tag{16}$$

The matrix S_{c1}, called the *scaling matrix*, is chosen as follows:

$$S_{c1} = k_{sc1} I, \qquad \exists\, k_{sc1} \tag{17}$$

for the simplicity of the design.

Similar to the (heave, pitch) system, we can design the control system for the (roll, warp) system.

B. Weighting Function

The three accelerations of the controlled value z_1 and z_2 express the ride comfort. Moreover, a human being is very sensitive to horizontal vibrations that occur over the frequency range 1–2 Hz and to the vertical vibrations over the frequency range 4–8 Hz and less sensitive to both of them that occur outside these frequency ranges, according to the *International Organization for Standardization* (ISO). On the other hand, the body resonant frequency of heave, roll, and pitch movements of our experimental vehicle is around 2 Hz. Therefore, we choose the weighting functions as shown in Fig. 3 for ride comfort.

If the body horizontal level is kept constant during running, driving is much easier. Therefore, it is desired that the suspension deflections are reduced over the whole frequency range. Then, we choose constant weighting functions for the suspension deflections.

C. Robustness Function

Due to neglected nonlinearities through linearization and so on, the model contains various kinds of uncertainties. In this paper, such uncertainties are regarded as unstructured uncertainties at the input channels, for example, the dead time of the actuator. Fig. 4 shows the normalized errors of the body accelerations between experiment and simulation and the robust function which is chosen to bound the errors.

D. Controller Design

An H^∞ robust controller to achieve these control objectives is obtained by solving the H^∞-control problem for the augmented plant $\overline{H}^{-1}$ of Fig. 2, which satisfies the norm condition of (16). In the actual design, the controller is designed independently for each decoupled subsystem of system (5). Since $D_{21} = 0$, the problem is not *standard* [19]. However, we can get the controller by assuming that D_{21} is small but nonzero. Similarly, the robust controller for the (roll, warp) system is designed.

V. Performance Evaluation

We carry out three kinds of tests by using four-liter commercial cars to evaluate the performance of the closed-loop system, that is, ride comfort and body leveling.

First, the performance of ride comfort for a stationary vehicle is evaluated by a *shaker test* which uses a four-wheel shaker. Second, the performance of ride comfort is evaluated by *road test* for a bad road. Third, the performance of body leveling is evaluated by *lane change test*, a level ground.

A. Shaker Test

Fig. 5 shows the frequency response with respect to the vertical acceleration against the heave mode road displacement.

In Fig. 5, although the vertical acceleration for heave mode road displacement is somewhat higher in the active case than in the passive one at the 3–7 Hz region, the active suspension rather improves the 2-Hz region of the body resonant frequency. This shows the effectiveness of the weighting function W_1 shown in Fig. 3.

B. Road Test (Bad Road)

Fig. 6 shows the power density spectrums (PSD) of the vertical acceleration for the vehicle running on a bad road at 50 km/h. Although it is somewhat worse on the active case than the passive one below 1 Hz, it is improved at the 2-Hz range of the body resonant frequency.

C. Lane Change Test

The performance of body leveling is important, especially for driver's handling. Fig. 7 shows the time response of the roll mode suspension deflection, namely, Z_{str} of (13), for the vehicle changing lane at 70 km/h. Apparently, the roll mode suspension deflection is much smaller for the active case than the passive one. Therefore, the active suspension improves the lateral body leveling significantly.

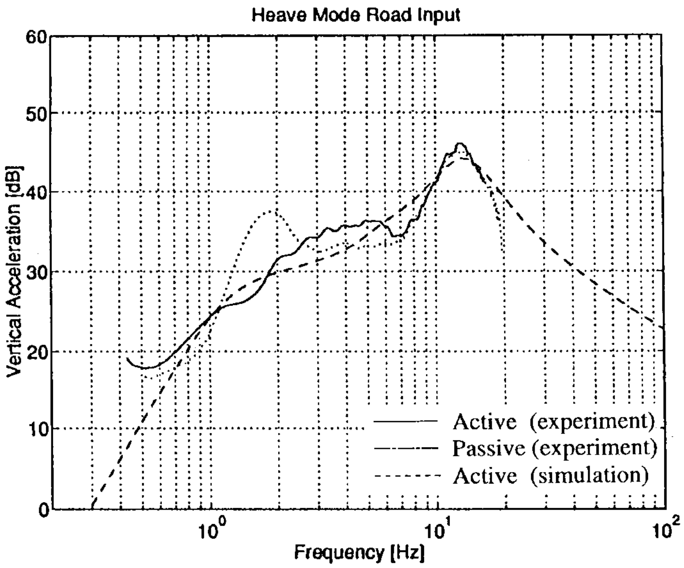

Fig. 5. Frequency responses of the body accelerations for the shaker test.

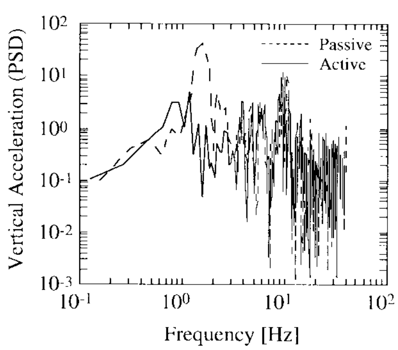

Fig. 6. Comparison of the body accelerations for the road test on a bad road between the active case and passive one.

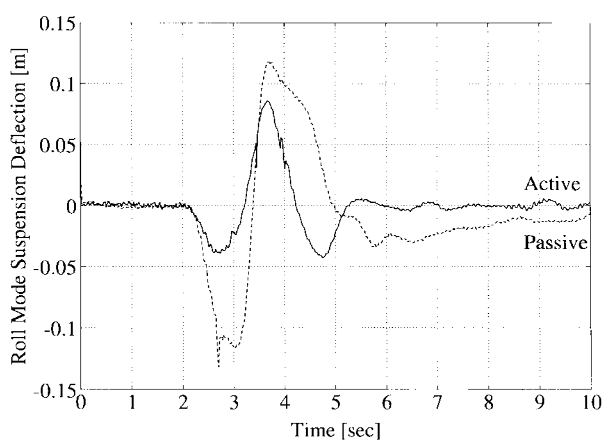

Fig. 7. Comparison of the roll mode suspension deflection for the lane change test between the active case and passive one.

VI. Conclusions

The full vehicle model for the purpose of an active suspension design has been derived systematically based on the Kane's dynamics. The decoupled structure of this model has been explored in order to make the implementation of the output feedback control simpler and realizable. The H^∞-control theory is applied to the design of an output feedback controller which takes the robustness with respect to the parameter uncertainty of the model into account. It has been demonstrated that this H^∞-output feedback controller is effective for the control of the active suspension system both by the simulations and the experiments.

Finally, the controller obtained has been actually implemented in a commercial car to evaluate experimentally the performance of ride comfort and body leveling. Three kinds of tests have demonstrated that the active suspension system proposed in this paper has improved ride comfort and body leveling simultaneously through an output feedback control which uses only a small set of easily available sensors. This shows the effectiveness of H^∞ control in which the robustness against parameter variations is explicitly taken into account for the control of active suspension systems.

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