Your name: G#:

George Mason University

Department of Electrical and Computer Engineering

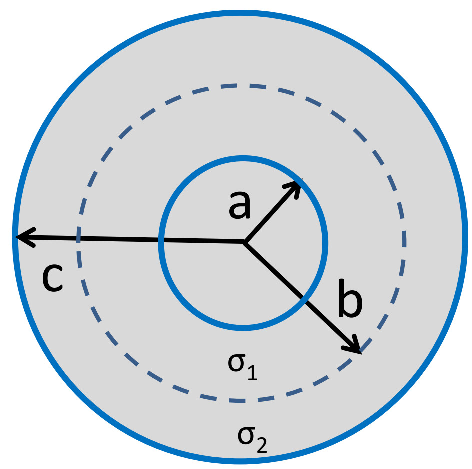
ECE 305 --- Electromagnetic Theory Fall 2019

Mid-term Exam #2 Date: Wednesday 11/15/2019

Policy: Provide details of the solution for each problem. A solution with only final results will not get credit.

Problem 1 (40 pts) Consider a spherical conductor with inner shell (r = a) and outer shell (r = c). The conductor is combined by two materials, conductivity = σ_1 for a < r < b and σ_2 for b < r < c. Please determine the resistance between these two shells (r = a and r = c). Do the following

(a) Assume boundary conditions
(b) Observe the symmetry and select the Laplace's equation to solve potential profiles. (separate it into two conductors in series)
(c) Determine the electric fields.
(d) Determine the current density and the current.
(e) Determine the resistance



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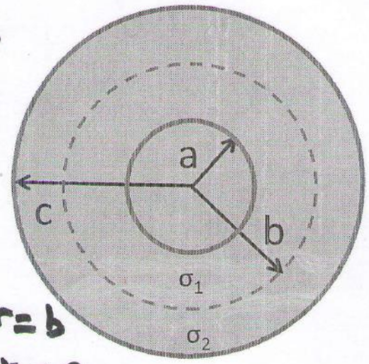

Solve: (a) separate the conductor into two spherical conductors

$\begin{cases} r=a \\ r=b \end{cases}$ and $\begin{cases} r=b \\ r=c \end{cases}$, in series

For Conductor : r=a to r=b

set boundary conditions $V = \begin{cases} 0, & r=b \\ V_0, & r=a \end{cases}$

(b) use Laplace's Eq. $\nabla^2 V = 0$

From spherical symmetry $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$

$\Rightarrow V = -\frac{A}{r} + B$.

use B.C.s $\Rightarrow A = -\frac{ab}{b-a}V_0$, $B = \frac{-a}{b-a}V_0$

(c) $\vec{E} = -\nabla V = -\frac{ab}{b-a}V_0 \frac{1}{r^2}\hat{a}_r$

(d) $\vec{J} = \sigma\vec{E} = -\frac{ab}{b-a}\sigma_1 V_0 \frac{1}{r^2}\hat{a}_r$. $I = \int \vec{J}\cdot d\vec{s}$

$\therefore I = -\frac{ab}{b-a}\sigma_1 V_0 \int \frac{1}{r^2} r^2 \sin\theta \, d\theta d\phi = \frac{4\pi V_0 \sigma_1 ab}{b-a}$

(e) $R_1 = V_0/I = (b-a)/4\pi\sigma_1 ab$ for Conductor $r=a \Rightarrow r=b$

so far, we determine the resistance of Conductor 1 from r=a to r=b

$$R_1 = \frac{b-a}{4\pi\sigma_1 ab}$$

similarly, the resistance of conductor 2 ($r=b \Rightarrow r=c$)

is $R_2 = \frac{c-b}{4\pi\sigma_2 cb}$

$\therefore$ Total resistance $R = R_1 + R_2$

$$R = \frac{b-a}{4\pi\sigma_1 ab} + \frac{c-b}{4\pi\sigma_2 bc}$$

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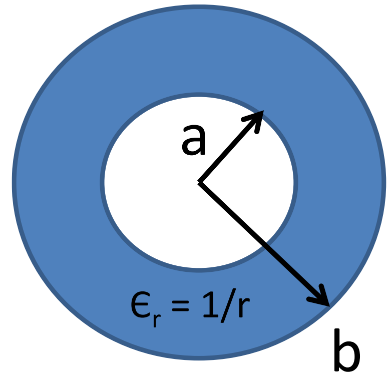
Problem 2 (40 pts): A spherical capacitor (see the figure below) has inner shell with radius ***a*** and outer shell with radius **b**. If the space between the shell is filled with inhomogeneous dielectric with $\epsilon_r = 1/r$, where variable r is the radial distance from the center to any observation point: **a** < **r** < **b**. Please determine the capacitance of the spherical capacitor.



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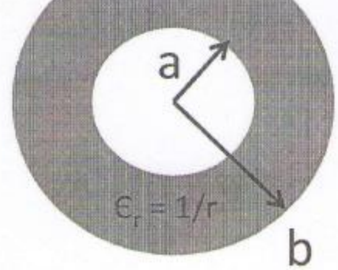

Solve:

use Gauss's Law

$$\vec{E} = \frac{Q}{4\pi \epsilon r^2} \hat{a}_r$$

- by assuming Q at sphere r=a and -Q at sphere r=b
- use spherical coordinate system

$$\therefore \vec{E} = \frac{Q}{4\pi \epsilon_0 r} \hat{a}_r \quad \text{for } a<r<b$$

Now, we need to find the Voltage difference between spheres r=a and r=b

$$V = V_a - V_b = -\int_a^b \frac{Q}{4\pi\epsilon_0 r} \hat{a}_r \cdot dr\, \hat{a}_r$$

$$= -\frac{Q}{4\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

$$\therefore C = \frac{|Q|}{|V|} = Q \Big/ \frac{Q}{4\pi\epsilon_0} \ln\left(\frac{b}{a}\right) = \frac{4\pi\epsilon_0}{\ln(b/a)}$$

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Problem 3 True or False (20 pts)

True	False	
(X)	( )	The potential difference between two points in an isolated conductor is zero.
( )	(X)	The resistance of any resistor is (resistivity × length) ÷ cross-section area
( )	(X)	The charge on any grounded conductor surface is zero.
(X)	( )	Mechanical pressure can be generated by flowing charges in an electric field
( )	(X)	For any dielectric interface between medium 1 and 2, $\mathbf{E}_{1t}=\mathbf{E}_{2t}$ and $\mathbf{D}_{1n}=\mathbf{D}_{2n}$