Your name: G#:

George Mason University

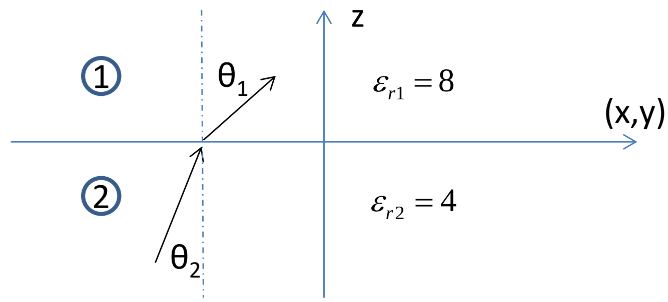
Department of Electrical and Computer Engineering

ECE 305 --- Electromagnetic Theory Fall 2019

Mid-term Exam 2 Date: Thursday 11/14/2019

Policy: Provide details of the solution for each problem. A solution with only final results will not get credit.

Problem 1 (40 pts) Two extensive homogeneous isotropic dielectrics meet on plane z=0. For z > 0, $\varepsilon_{r1} = 8$ and for z < 0, $\varepsilon_{r2} = 4$. A uniform electric field $\mathbf{E}_1 = 5\mathbf{a}_x - 4\mathbf{a}_y + 3\mathbf{a}_z$ V/m exists for $z \geq 0$. Find (1) $\mathbf{D}_2$ for $z \leq 0$; (2) what are the values of angle θ_1 and θ_2 ?



problem #1

$$\vec{E}_1 = 5\hat{a}_x - 4\hat{a}_y + 3\hat{a}_z \quad V/m$$

$$\therefore \vec{D}_1 = 8\varepsilon_0(5\hat{a}_x - 4\hat{a}_y + 3\hat{a}_z)$$

Refer to xy-plane with normal direction @ z-axis

$$\vec{E}_{1t} = 5\hat{a}_x - 4\hat{a}_y \quad V/m$$

$$\vec{D}_{1n} = 24\varepsilon_0\hat{a}_z$$

For the Boundary at xy-plane. $\rho_s = 0$

$$\therefore \vec{D}_{2n} = \vec{D}_{1n} = 24\varepsilon_0\hat{a}_z$$

and $\vec{E}_{2t} = \vec{E}_{1t} = 5\hat{a}_x - 4\hat{a}_y \quad V/m$

$$\therefore \vec{D}_{2t} = \varepsilon_{r_2}\varepsilon_0\vec{E}_{2t} = 4\varepsilon_0(5\hat{a}_x - 4\hat{a}_y)$$

(1) $\vec{D}_2 = \vec{D}_{2n} + \vec{D}_{2t} = 20\varepsilon_0\hat{a}_x - 16\varepsilon_0\hat{a}_y + 24\varepsilon_0\hat{a}_z$

(2) To find angle θ_2

$$\vec{E}_2 = \vec{D}_2/\varepsilon_{r_2}\varepsilon_0 = 5\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z \quad V/m$$

$$\tan\theta_2 = \frac{|E_{2t}|}{|E_{2n}|} = \frac{\sqrt{25+16}}{\sqrt{36}} = \frac{\sqrt{41}}{6} \approx 46^\circ$$

$\theta_2 = 46^o$

similarly, it can be calculated as $\theta_1 = 65^o$

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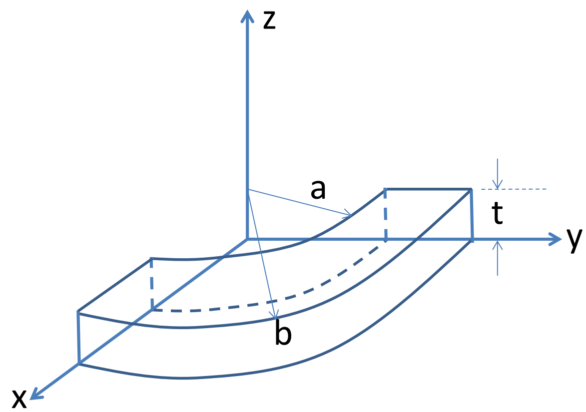
Problem 2 (40 pts): A partial cylindrical conductor with conductivity σ.

(a) Please determine the resistance between the surfaces at $z = 0$ and $z = t$.

(b) Please determine the resistance between the cylindrical surface between $\rho = a$ and $\rho = b$.



Problem #2

(a) Because the cross-section along z-axis is homogeneous

$$R_z = \frac{t}{\sigma \cdot S}$$

where $S = \frac{1}{4}(\pi b^2 - \pi a^2) = \frac{\pi}{4}(b^2 - a^2)$

$$\therefore R_z = \frac{4t}{\pi\sigma(b^2 - a^2)}$$

(b) assume $V = V_0$ at $\rho = a$, $V = 0$ at $\rho = b$

use Laplace's Eq. at cylindrical coordinate

$$\frac{1}{\rho}\left(\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right)\right) = 0$$

$$\therefore \rho\frac{\partial V}{\partial\rho} = A \Rightarrow V = A\ln\frac{\rho}{\rho_0}$$

where A and ρ_0 are constant T.B.D.

use assumption $V = V_0$ at $\rho = a$ and $V = 0$ at $\rho = b$

$$\begin{cases} V_0 = A\ln\frac{a}{\rho_0} \\ 0 = A\ln\frac{b}{\rho_0} \end{cases} \Rightarrow \begin{cases} A = V_0/\ln\frac{a}{b} \\ \rho_0 = b \end{cases}$$

$$\therefore \vec{E} = -\nabla V = -\frac{A}{\rho} = -\frac{1}{\rho}V_0/(\ln a/b)$$

$$\therefore \vec{J} = \sigma\vec{E} = \frac{\sigma V_0}{\ln b/a}\frac{1}{\rho}$$

current $I = \vec{J}\cdot\vec{S}\,\Big|_{\text{at }\rho=a} = \frac{\sigma V_0}{\ln b/a}\cdot\frac{1}{a}\cdot\frac{\pi}{2}a\cdot t$

$$= \frac{\pi\sigma t V_0}{2\ln b/a}$$

$$\therefore R_\rho = \frac{V_0}{I} = \frac{2\ln b/a}{\pi\sigma t}$$

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Problem 3 True or False (20 points)

True	False	
(X)	( )	The potential difference between two points in an isolated conductor is zero.
( )	(X)	The resistance of any resistor is (resistivity × length) ÷ cross-section area
( )	(X)	The charge on any grounded conductor surface is zero.
(X)	( )	Mechanical pressure can be generated by flowing charges in an electric field
( )	(X)	For any dielectric interface between medium 1 and 2, $\mathbf{E}_{1t}=\mathbf{E}_{2t}$ and $\mathbf{D}_{1n}=\mathbf{D}_{2n}$