Your name: G#:

George Mason University

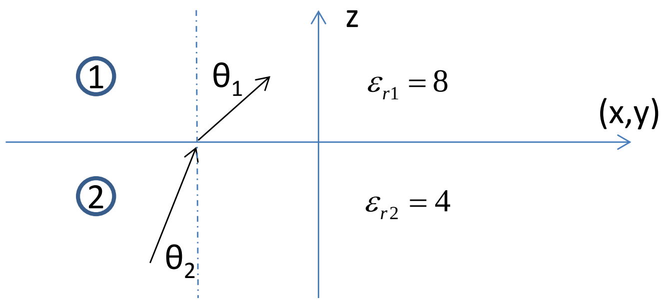
Department of Electrical and Computer Engineering

ECE 305 --- Electromagnetic Theory Fall 2016

Mid-term Exam 2 Date: Monday 11/08/2016

Policy: Provide details of the solution for each problem. A solution with only final results will not get credit.

Problem 1 (30 pts) Two extensive homogeneous isotropic dielectrics meet on plane z=0. For z > 0, $\varepsilon_{r1} = 8$ and for z < 0, $\varepsilon_{r2} = 4$. A uniform electric field $\mathbf{E}_1 = 3\mathbf{a}_x + 4\mathbf{a}_y + 5\mathbf{a}_z$ V/m exists for $z \geq 0$.
Find (a) $\mathbf{E}_2$ for $z \leq 0$;
(b) The angles $\mathbf{E}_1$ and $\mathbf{E}_2$ make with the interface.
Solution: (check example 5.9)

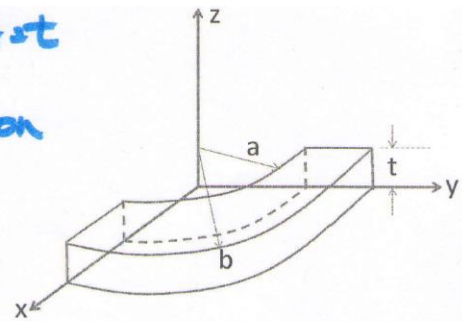


Problem 2 (40 pts): A partial cylindrical dielectric with dielectric constant: $\epsilon_r = 6$.

(a) Please determine the capacitance between the surfaces at z = 0 and z = t.

(b) Please determine the capacitance between the cylindrical surface between ρ = a and ρ = b.

Solution: (check example 6.8)



(a) Capacitance b/w z=0 and z=t

b/c of uniform cross-section

$\because C = \frac{\varepsilon S}{d}$

where $\varepsilon = \varepsilon_r \varepsilon_0 = 6\varepsilon_0$

$S = \frac{1}{4}(\pi b^2 - \pi a^2)$

$\therefore C = \frac{3\pi\varepsilon_0 (b^2 - a^2)}{2t}$. [or you can use Laplace's equation ....]

(b) For capacitance b/w surfaces at ρ = a and ρ = b we must use Laplace's equation

$\nabla^2 V = 0 \Rightarrow \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) = 0$

$\therefore V = A\ln(\rho) + B$

Set B.C.s $V = \begin{cases} 0, & \rho = a \\ V_0, & \rho = b \end{cases}$

$\therefore A = \frac{V_0}{\ln(b/a)}, \quad B = -\frac{V_0 \ln(a)}{\ln(b/a)}$

$\therefore \vec{E} = -\nabla V = -\frac{A}{\rho}\hat{a}_\rho$

Find Q at ρ = a: $Q = \int_s \varepsilon \vec{E}\cdot d\vec{s} = 6\varepsilon_0 \int \frac{A}{\rho}\rho\, d\phi\, dz$

$\therefore Q = \frac{3V_0\pi t\varepsilon_0}{\ln(b/a)} \Rightarrow C = \frac{Q}{V_0} = \frac{3\varepsilon_0\pi t}{\ln(b/a)}$

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Problem 3 (30 pts): In cylindrical coordinates, V = 0 at ρ = 10 m and V = V_0 at ρ = 30 m. If $E = -6\boldsymbol{a_\rho}$ kV/m at ρ = 20 m, determine V_0.

Solution: (check homework problem 6.12)